Norwegian School of Management BI

Final Master Thesis

Industry Dynamics and Productivity: The E ect of Productivity Change

Abstract

The purpose of this study is to investigate to what extent it is the most productive rms who attract workers. Using the microeconomic models of general equilibrium, Cournot and Hotelling competition, search models and related empirical studies we postulate an econometric model. We use a unique dataset on Norwegian manufacturing rms from the years 2000 to 2008. We nd that more productive rms have a higher average annual worker growth. This does not necessarily mean that more productive rms are larger. However, given enough time a faster growing but small rm is expected to be larger than a large but slow growing rm. We also nd that rm growth decreases with size, rejecting Gibrat's Law. Our ndings give suggestive evidence to the theories of competitive search models, which state that more productive rms o er higher wages, have more vacancies and attract workers faster. These results survive several robustness checks, including alternative productivity measure and an alternative structural form. In addition, we nd that our data con rms a collection of stylized facts often found in the literature.

Preface

This paper is written as a Master of Science thesis at BI Norwegian School of Management. We would like to thank our supervisor Espen Moen for valuable guidance, counseling and exibility. He provided us with the initial topic for this thesis. We would also thank the Center of Corporate Governance Research for providing the data used in our study.

Barcelona, 9th of August, Jonas Momkvist and yvind Nilsen Aas.

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1 Introduction

We investigate the relationship between productivity and worker reallocation of Norwegian manufacturing rms. Understanding the determinants of worker reallocation is important since reallocation of resources as a result of creative destruction, is an important factor in economic growth [57] [6] [45]. Advances in productivity, that is the ability to produce more with the same or less input, is a signi cant source of increased potential national income [47]. The process of creative destruction leads to job destruction and creation which leads to shifts in resources. The process is driven by di erent types of innovations and improvements, which could be summarized as productivity [57].

The purpose of this study is to investing to what extent it is the most productive rms who attract workers. In more speci c terms, our research question is *"How is the ow of workers a ected by productivity?"* We look at the microeconomic models of general equilibrium, Cournout and Hotelling competition and competitive search. These models predict that more productive rms will attract more workers. We then postulate an econometric model based on the theory and related empirical literature. We nd that more productive rms have a higher average annual growth rate, meaning that they attract more workers. This nding does not necessarily mean that the more productive is larger. However, given su cient time a small fast growth rms will be larger than a larger slow growing.

According to OECD the world economy have experienced a formidable economic growth since the industrial revolution [47]. Joseph Schumpeter presented the ideas that economic growth is not necessarily driven by competition, but by creative destruction. In Schumpeter's *Capitalism, Socialism and Democracy* the author postulates a theory of capitalism which is *"by nature a form or method of economic change and not only never is but never can be stationary."* ([57], p. 82). According to Schumpeter, the driving force of economic growth and increasing standard of living in the capitalist system is the process of creative destruction. Creative destruction is industrial mutation that constantly destroys the old structure and instantly creates a new one.

Our main focus is on changes in employment demand, so our econometric strategy is to estimate the growth-productivity relationship using regression estimation. The elasticities of interest are those of productivity, wage, size and age. In addition, we estimate a collection of stylized facts often found in the literature. We take advantage of a unique panel data set of 2 942 manufacturing rms in Norway for the years 2000 to 2008. Our

2 De nitions and related literature

The purpose of this section is to present the terminology of our paper, related studies and

Following these notations, we can write job ows as

$$JF_{it} = H_{it} \quad D_{it} = S_{it} \quad S_{it \ 1} \tag{5}$$

of rms, the proportional rate of growth is smaller according as the rm is older, but its probability of survival is greater. Several studies [28], [34], [25], have found

11.3 percent. The heterogeneity is a result of large rates of job creation and job destruction.³ Haltiwanger [35] found that worker ows are closely connected to rm outcomes, re ecting in large part the ongoing shift in resources from less productive to more productive employers.

6. Entry and exit of plants with di erent productivity levels is an important source of productivity growth. A large portion of aggregate productivity growth can be attributed to resource reallocation. The manufacturing sector is characterized by large shifts in employment and output across establishments every year. These large shifts are a major force contributing to productivity growth, resurrecting the Schumpeterian idea of creative-destruction [17]. John Baldwin explains the pattern of productivity and output as the following. In general, entrants are smaller than the average incumbent, and about half die within the rst decade. If the entrant survive, they reach average productivity in about a decade, they are however still smaller than the average rm. Essentially the pattern is survival of the ttest, the process of weeding out the unsuccessful entrants and nurturing the successful ones [8].

³This is also found in Albaek and Sorensen [2] and Burgess et all. [15]

3 Microeconomic theory

In this section we will review the general equilibrium model, Cournot and Hotelling competition and search theory. The predictions of these models is the basis of our hypothesis.

3.1 General equilibrium

Consider an economy with perfect competition. Each rm is a small player in the industry. The price of the product is una ected by the quantity of output produced by the individual rm, and the price of inputs are also una ected by the individual rm's factor demand. All products and inputs are homogeneous. We will rst address the optimal decisions by the rms in short term and later in long term.

By short term, we mean a su ciently short period such that capital is xed. Consider a Cobb-Douglas production function with two inputs and decreasing returns to scale in labor⁴. The rms set labor to minimize the following cost function given the levels of

and the short term marginal cost thus becomes

$$\frac{d}{dq}(C(W; r; q)) = C^{\theta}(W; r; q) = \frac{W}{AK} \frac{q^{1}}{AK}$$
(9)

namely the long run cost function with is derived in the following way [63].

$$C(w; r; q) = \min_{K; L} r \quad K + w \quad L$$

such that $A_i K \quad L^1 = q$

Where $A_i > 0$ is a level speci c production technology. Solving the optimization we obtain the optimal demand for K and L

$$\mathcal{K} = q \quad \frac{1}{A_i} \quad \frac{W}{1 \quad r} \quad \frac{1}{r} \tag{12}$$

$$L = q \quad \frac{1}{A_i} \quad \frac{1}{w}$$
(13)

The cost function is de ned as:

$$C(W; r; q) = r \quad K \quad + W \quad L \tag{14}$$

Combining (12), (13) and (14), yields the following cost function

$$C(w;r;q) = \frac{1}{A_i} \prod_{i=1}^{n} \frac{1}{1} + \frac{1}{1} \prod_{i=1}^{n} \frac{r}{w^1} q \qquad (15)$$

$$C^{\emptyset}(W;r;q) = \frac{1}{A_i} + \frac{1}{1} + r^{-1}$$
(16)

We see from equation (16) as A_i increases the marginal cost will go down. As a result, the rms with the highest productivity level, will have the lowest marginal cost. The rms optimize in the same way as in the short run, solving Equation (10), setting price equal to marginal cost given by

$$p = \frac{1}{A_i} \left[\begin{array}{ccc} & & & & \\ & 1 & & \\ 1 & & & \\ \end{array} \right]^1 + \frac{\#}{1} r w^1$$
(17)

In short run, rms with di erent productivity levels coexisted. According to Equation (17) the equilibrium price for the rms on di erent levels will vary. The rms on the level with the highest productivity, will have the lowest marginal cost and therefore the equilibrium price will be lowest. The rms which produce at the lowest marginal cost (highest A_i) will be the only ones selling the product and the only ones that want to allocate more capital and labor. As a result only the most productive rms will stay and the less productive will exit. Since the total number of rms in the economy is reduced,

the remain rms are larger relative to when there where many level. Since the optimal labor demand is a function of both A and q, (d=dA)(q) > 0 there exists a trade o for the rm. An increase in productivity increases optimal quantum produced (quantum e ect) and reduces the workers needed since the rms are more productive (utilization e ect). The total e ect will depend on which of these two e ects are the dominating.

Using the notion of di erent productivity levels the rms with productivity level A_1 are the ones which will survive. Let A_1 A_2 = and let > 0. The rms with level 1 will always undercut the rms with level 2. However, since there is su ciently many rms on each level, the price will still be equal to marginal cost. The result is that only rms with the same productivity level can exist in the market.

In conclusion, in the short term there can exist rms with di erent *AK* levels, since there is decreasing returns to scale due to xed capital. In the long run, only the most productive rms will produce, such that all rms in the economy have the same productivity level. If there is a di erence in long run, there will be a reallocation of inputs from the low productivity rms to the high productivity rms. A productivity change leads to a reallocation of inputs from the low productive rm is expected to have a larger work force.

3.1.1 Adjustment cost

In the model of general equilibrium rms immediately adjust their capital and labor when productivity changes, leading to an instant ow of inputs. However, in the real economy there is considerable lag in demand for inputs [37]. One explanation for the observed phenomenon could be adjustment costs related to changes in input. In the Cobb-Douglas production function there may be adjustment costs related to changes in the work force and capital.

The literature investigating adjustment costs has two approaches, namely convex and non-covex costs of adjustment. Holt et all [38] found a quadratic speci cation of adjustments costs to be a suitable rst approximation in certain industries. To avoid the increasing costs the rm will adjust their input often by small amounts, causing distributed lags [5]. According to Doms-Dunne [24] non-convex cost of adjustment focus either on xed or proportional costs of adjustment, making characteristics of optimal behavior hard to outline. The implications may be certain number of periods without adjustments, and at selected times sizable adjustments [5]. These implications are contradicting to those of the quadratic adjustment cost which yield small and continuous

adjustments.

Labor adjustment costs will directly a ect labor demand. A productivity shock may create a lag in convergence to its new long run equilibrium if there is convex adjustment cost. But if the the adjustment costs are non convex there may be a immediate jump to the new long run equilibrium, or the rm can maintain its old employment level if the shock is not large enough [36]. The cost related to capital adjustment play an important role in determining the labor demand. If a positive productivity shock occur, the rm demand more of both inputs. If the adjustment cost of capital is convex there will be a slow transition towards the long run equilibrium level of capital. On the other hand if there is a non-convex adjustment cost, adjustments occur as a jump.

In conclusion, convex adjustment cost may create a lag in convergence to the new long run equilibrium after a productivity shock. However if adjustment cost are non-convex there may be a immediate jump to the new long run equilibrium, or unchanged behavior if the the shock is not large enough [36].

3.2 Cournot

Consider an economy with a nite number of homogeneous rms competing in the nal goods market, and in nite many agents supplying the input factors. There is free competition in the input market, such that all prices are marginal prices. In the nal good, the rms compete on quantity, here represented by a repeated game of Cournot with in nite many periods, or uncertainty about when the last period will be. All agents maximize the pro t function in Equation (18), taking into consideration the other rms actions. Consider a symmetric case with linear demand and a Cobb-Douglas cost function C(w; r; q) yields the following pro t function⁵

$$i = q_i (1 \quad q_i \quad q_j) \quad c_i \quad q_i$$
 (18)

where c_i is the unit cost for the *i*-th rm, de ned as

$$C_i = A_i^{1} \frac{1}{1} + \frac{\#}{1} r w^1$$
 (19)

where q_i is the quantity produced by rm i, q_j the quantity by rm $j \in i$, A_i is a rm speci c production technology, is the capital share, and r and w are the input prices.

⁵See Appendix A for the derivations of the model

By taking the rst order conditions w.r.t. q_i and q_j , and solving the reactions functions with respect to the optimal action by the other rm we get that

$$q_i = \frac{1 - 2c_i + c_j}{3}$$
(20)

$$q_j = \frac{1 - 2c_j + c_i}{3}$$
(21)

Equation (20) show the optimal quantum produced by rm *i* to be a function of the marginal cost of its own production and its competitor. Firm *i* which minimizes the cost of production for a given quantity has the following factor demand

$$K = q_i \frac{1}{A_i} \frac{w}{1 + r} = q_i \frac{1}{A_i} \frac{1}{r} \frac{r}{w}$$
 (22)

Where demand for inputs is a function of the quantity produced, the rm speci c technology, factor intensities and the price of the two inputs. It is important to note that by Equation (20), q_i is a function of the technology parameter as well. This relationship will have a profound e ect on the demand when productivity change is introduced.

After the rst period is over, right before the next period starts, the rms may experience a productivity shock, such that $A^{\ell} \notin A$. Resulting either in the rm becoming more productive or less productive. The probability of experiencing a productivity shock is non-negative for all rms. By looking at the derivative of Equation (20) and (22) with respect to the technology, A_i , we can de ne what the theory suggest is the e ect of productivity change.

$$\frac{@q_i}{@A_i} = \frac{2}{3} \frac{1}{A_i^2} \qquad \frac{1}{1} \qquad + \qquad \frac{\#}{1} \qquad r \ w^1 > 0$$
(23)
$$\frac{@L}{@A_i} = \boxed{\bigcirc}_{@} \frac{2}{3} \frac{1}{1}$$

(24) which of these e ects are the strongest because it depends on the parameter , the input prices r; w and the quantity produced, q.

The Cournot model predicts that if a rm experiences a positive productivity shock there will be a change in inputs, but the exact sign is not clear. However, there is a link between productivity and input allocation. In addition, we see rms with di erent productivity levels coexisting in the economy under the condition that the di erence in productivity is su ciently small. There exists an interval A_i $A_j = ", (A_i 6$ where $p_2 > p_1$. The respective market shares of the two rms then become

$$x^{m} = \frac{1}{2} + \frac{c_{2} - c_{1}}{6t}$$

$$(1 - x^{m}) = \frac{1}{2} + \frac{c_{1} - c_{2}}{6t}$$

$$(28)$$

We see that if the two rms where identical $(c_1 = c_2)$, the result would be to split the market. However, since $c_1 < c_2$ rm 1 obtains a larger market share than rm 2. These are the optimal market shares from the two rms point of view. However, this is not necessary what is optimal for society. We look at the objective function for society as a whole

$$W = V \quad c_1 \quad y \quad c_2(1 \quad y \quad) \quad \frac{t \quad y \quad ^2}{2} \quad \frac{t(1 \quad y \quad)^2}{2}$$
(29)

By maximizing the social wealth function W, with respect to y we nd the socially optimal market shares

y =

are treated as endogenous variables. These models provide a richer equilibrium framework in contrast to the frictionless competitive models [52]. The most used model of wage

where $k_0 = 0^{-1}$

persion because o ering a wage equal to a mass point is not pro t maximizing in the sense of equation (35).

A critical feature of the model is the positive relationship between the wage o er and employers labor force size it implies. As the voluntary quit rate, ${}_{1}F(w)$, decreases with the wage o er, larger rms experience lower quit rates. Because workers only switch employers in response to a higher wage o er, workers with either more experience or tenure are more likely to be earning a higher wage.

3.4.2 Job Productivity Di erentials

We will now look at what happens if we introduce heterogeneity among employers, specifically two types of employers. One of the employers is more productive then the other and earn a higher revenue ow per workers such that $p_2 > p_1$. The fraction of employers of type 2 is denoted . The model is identical to the one above in all other aspects such that an equilibrium can be described by $(F_1; F_2; R; 1; 2)$, where the reservation wage satis es equation (31) and $F_1; F_2$ represent an o er distribution of the two types of employers and

(р

(prea38]TJ/F19 11.91(2) -442(is) -441(denot) -44]TJ/F.

and F_2 can be written as

$$F_{i}(w) = \frac{k_{1}}{1+k_{1}} 41 \qquad \frac{p_{i}}{p_{i}} \frac{w}{p_{i}} 5$$
(41)

on its support $[\underline{W}_i; \overline{W}_i)$, i=1,2.

$$\underline{w}_i = R; \text{ where } R \text{ satis } es (31)$$

$$\overline{w}_1 = \underline{w}_2; \text{ where } p_1 \quad \overline{w}_1 = (p_1 \quad \underline{w}_1) = (1 + k_1)^2$$

$$p_2 \quad \overline{w}_2 = (p_2 \quad \underline{w}_2 = (1 + k_1)^2$$

4 Method and Approach

In our paper we will take an empirical approach. The combination of microeconomic general equilibrium, Cournot, Hotelling, search theory and empirical studies of rm dynamics are used to identify factors a ecting worker allocation between rms. The theory driven model will be estimated using data from the Center for Corporate Governance Research (CCGR). The CCGR dataset is an unbalanced panel containing accounting data for Norwegian rms with limited liability in the period 1994 to 2008. The scope of our investigation will be the manufacturing industry, classi ed according to the OECD NACE codes. A lot of the research on productivity and rm dynamics are conducted on the manufacturing industry, which makes it easier to relate to previous literature.

The econometric estimation will start with an investigation of well-established stylized facts. The reason for this query is to see if our data have similar properties to those found in previous studies. Next, we estimate our model using OLS regression analysis. In addition, we test the results for misspeci cation, heteroskedasticity and mulitcollinearity to determine the most e cient estimator. Lastly we will run a least square dummy variable (LSDV) to allow for the intercept to vary across di erent sectors of the manufacturing industries. We end our empirical analysis with robustness tests on our estimates using an alternative measure of productivity and an alternative structural form. All econometric tests and regressions are performed using STATA 9.0 and STATA 10.

 m_{it} is material cost, w_{it} is the average wage rate, r_{it} is capital cost, q_{it} is the quantum produced for each rm, is the capital share, Age_{it} is the age of the rm, S_{it} is number of employees and it

Alternatively, we could have modeled the relationship between rm size in period t (S_t) and F(

at the relationship between bankruptcy risk and age and size.

The rst data set containing 69 368 annual observations or 13 799 rm observations is cleaned in the following way. First we need a balanced data set, so we delete those rms that are not present in the whole period, which removed 10 547 rms. The removal of rms which go bankrupt or disappear reduces much of our variability and could leave to survival bias. This potential problem is discussed more thoroughly after the results are presented. We have not controlled for this problem in our estimation. In order to estimate our equation we need the variables on log form. When creating logs we remove those 310 observations that become missing values due to the log operation. Now we have 2 942 rm observations or 26 478 annual (9 years) observations.

This second data set is cleaned in the following way. We rst need to nd out which rms that is present in the beginning of the period and the end. We start with 69 368 annual observations or 13 799 rm observations. We then remove all that are not present in 2000. This removed 5 742 rm observations. We create a survival variable which is coded 1 if they are present in 2000 and 2008 and 0 if they are present in 2000 but not in 2008. We generate the variables by taking log of employment and age. We now have 8 057 rm observations.

Due to the extensive screening process, we do not have a random sample. Our bal-

We have obtained the risk free interest rate as the 3 year government bond issued by

from our regression. Our equation in its nal form will be the following.

$$[\ln S_{it^{0}} \quad \ln S_{it}] = d = {}_{1} + {}_{2} \quad \ln A_{it}$$

$$+ {}_{3} \quad \ln W_{it} + {}_{4} \ln Age_{it} + {}_{5} \quad \ln S_{it} + {}_{it}$$
(51)

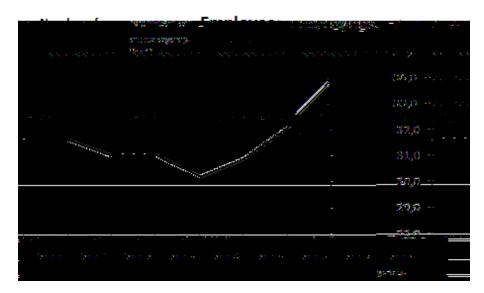


Figure 1: Employment Notes.

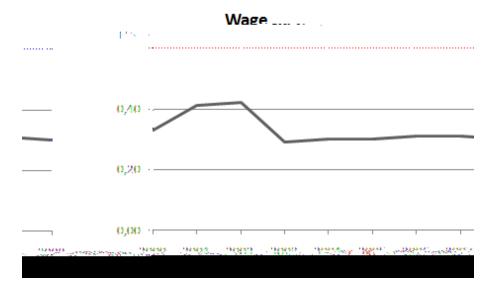


Figure 3: Wage. *Notes.* The gure shows the mean value for the average wage per employee.

5.5.2 Dispersion in productivity levels

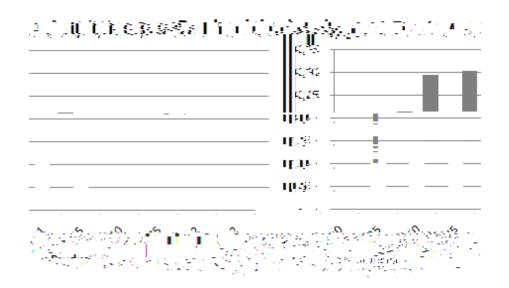


Figure 5: Productivity level. *Notes.* The gure show the percentage of rm of the total sample which lies in a particular productivity level interval. The y-axis is the percent of companies of the total sample. The x-axis is the productivity level intervals

The bar chart of the productivity dispersion is illustrated in Figure 5. The chart has intervals of value added per worker on the x-axis and percentage of rms from the total sample on the y-axis. We see that approximately 30 percent of our rms lie in the productivity interval 0:25 0:50. Meaning that 30 percent of our companies have a value-added which lie between 250 000 and 500 000. We also see that approximately 85 percent of our rms lie in the productivity interval 0 1. Furthermore, the bar chart shows that there is some dispersion in productivity levels. This is in line with the stylized facts in our literature review.

5.5.3 Productivity and wage

The scatter plot of the relationship between productivity and wage is shown in Figure 6. We use the mean of each rm's productivity on the x axis and mean wage on the y-axis to illustrate the relationship. We interpret the scatter plot as displaying a positive relationship between productivity and wage. As the mean productivity for a rm increases, the wage seems to increase. The plot does not show this relationship accurately because some rms have a low mean productivity and pay high wages, but the overall trend from the graphical illustration suggest a positive relationship. This is also supported by stylized facts found by Oi et all. [54]

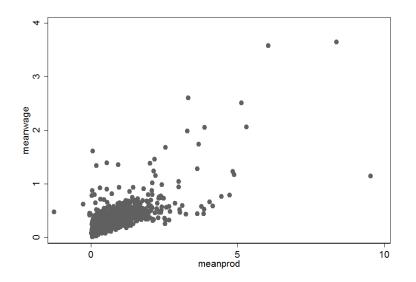


Figure 6: The relationship between mean wage and mean productivity *Notes.* The gure shows a scatter plot of mean wage on the y-axis and mean productivity on the x-axis.

5.5.4 Survival

Jovanovic's [42] theory of rm growth states that as time progresses, the rm will uncover their true e ciency level. Based on this e ciency level, they are able to evaluate the prospect value of remaining in business. If this value is negative, the rm will chose to go bankrupt or be dissolved. As part of our investigation of the stylized facts, we want to see how the probability of survival is a ect by the rm age and size. The probability of survival-variable is based on whether the rm is present in the beginning period and in the ending period. If the rm survives (present all years) we code the survival-variable with I = 1 and if the rm is only present in the beginning and not in the end (dissolved) we code with I = 0. According to Evans [28] the regression can be represented by the following equation.

$$E[I_{j}A_{it}; S_{it}] = Pr[e_{it} > V(A_{it}; S_{it})] = [V(A_{it}; S_{it})]$$
(52)

"where V can be though of as the value of remaining in business, e_t is a normally distributed disturbance with unit variance and is the cumulative normal distribution function with unit variance" ([28], p. 573). We take a rst-order logarithmic expansion of the growth function and estimated our equation using probit regression. According to Cameron and Trivedi [16], there is little di erence between a logit and a probit model when the focus is on the marginal e ects at the mean of the sample. According to Amemiya [3] Equation (52) estimated with a maximum likelihood estimator will be consistent. In addition we adjust the error terms according to White [64].

Dependent variable	Survival	Variability of Growth
Size	0.1577* [0.0107]	0.7697* [0.0130]
Age	0.0486** [0.0158]	-0.1528* [0.0192]
Constant	-0.3470* [0.0647]	-1.2477* [0.0517]
Observations	8057	2942

Table 2: Firm Survival and Variability of Growth

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*Signi cant at the 1 percent level. **Signi cant at the 5 percent level

Partial Derivative of with Respect to ^a		Survival	Variability	
Size	Mean	0.0627	0.7697	
	Standard Deviation	0.0042	0.0129	
Age	Mean	0.0193	-0.1527	
	Standard Deviation	0.0063	0.0192	

Table 3: The E ect of Firm Size and Age on Firm Dynamics

^aPartial derivative of the regression function on the horizontal with respect to the logarithmic value of the variable on the vertical.

expansion and w is the error term. We use maximum likelihood and white adjusted error terms. The results from the estimation are presented in Table 2.

The results from estimation should be viewed with caution. The dependent variable is based on four growth observations and may be imprecise. The results shows that the variability of growth increases with size and decreases with age. At the sample mean, a 1 percent increase in size leads to 0.77 percent increase in the standard deviation of rm growth. A 1 percent increase in age leads to a 0.1527 percent decrease in the standard deviation of rm growth. The second result is in line with earlier studies by Evans [28] and Sutton [61].

5.6 Results

This section outlines the statistical tests and estimations. We will, as discussed in Section 5.1, estimate the relationship between worker reallocation and productivity using the following equation

$$[\ln S_{it^{0}} \quad \ln S_{it}] = d = {}_{1} + {}_{2} \quad \ln A_{it} + {}_{3} \quad \ln w_{it} + {}_{4} \ln Age_{it} + {}_{5} \quad \ln S_{it} + {}_{it}$$
(54)

This regression equation is estimated using the balanced panel dataset and ordinary least square (OLS) as estimator. The problem of autocorrelation is not present in our data since we only use data from the base year, 2000, as explanatory variables. Heteroskedasticity can be caused by omitted variables, incorrect functional form or skewness in the distribution of regressors [33]. We decide to check for heteroskedasticity by running the Breusch-Pagan-Godfrey (BPG) test [13] [32].

The BPG tests the null hypothesis of homoskedasticity. It is a Chi-Square test based on an auxiliary regression. This implies that the null hypothesis is rejected if the chisquare exceeds the critical chi-square at the given level of signi cance. This translates into the decision rule; reject the null hypothesis if the p-value of the test is below the signi cance level. We reject the null hypothesis of homoskedasticity (P < 0.05) and conclude that we have a problem of heteroskedasticity.

Notice that in presence of heteroskedasticity the OLS estimators are still linear and unbiased as well as consistent, but they are no longer e cient (i.e. minimum variance) [33]. As a consequence the OLS estimators is not the best linear unbiased estimator (BLUE). The problem of heteroskedasticity is a serious potential problem and one cannot rely on the conventionally computed con dence intervals and the t-test, f-test and chi-square test may not be valid. Hence it should not be used for conclusions or inferences because they might be misleading [33].

There may also be multicollinearity in our estimation. The presence of high multicollinearity give large variance and covariance, making precise estimation di cult. Furthermore, multicollinearity increases the probability of accepting the null hypothesis. It has, however, no e ect on the properties of the estimator.

We used a multicollinearity indictor, the VIF test¹⁵, to see if we have a problem of multicollinearity. The VIF test is a measure of collinearity. The larger the VIF value, the more collinear are the variable. A rule of thumb is a VIF value exceeding 10 indicate high collinearity [33]. All of our VIF values are less than 2.32, hence multicollinearity does not seem to be a severe problem in our data.

In addition, we run the Ramsey's regression speci cation error test (RESET). The Ramsey test is a general test of speci cation error [33]. The RESET tests the null hypothesis that the model has no speci cation error, i.e has no omitted variables. The test statistic reject the null hypothesis, no mis-speci cations (P < 0.01). Hence our model has, according to the RESET, omitted variables. The theoretical models in Section 5.1 suggest an econometric model adding rental rate of capital, capital share and output to the estimated equation. This could yield more precise estimates. Due to lack of data we are

¹⁵See Table 9 in Appendix C

Variable	(1)	(2)	(3)
Productivity	0.0454* [0.0034]	0.0226* [0.0048]	0.0238* [0.0017]
Wage		0.0361* [0.0087]	0.0367* [0.0030]
Age		-0.0119* [0.0020]	-0.0111* [0.0006]
Size		-0.0149* [0.0015]	-0.0158* [0.0005]
Cons.	0.0363 [0.030]	0.1277 [0.0104]	0.1380 [0.0046]
Industry dummies			Yes
Adjusted R ²	0.1401	0.2165	0.2373

Table 4: Regression results

Dependent variable: Annual change in workers, in logs. (1) OLS, (2) OLS (3) Fixed e ects. All standard errors are White-adjusted. *Signi cant at the 1 percent level. **Signi cant at the 5 percent level

unable to compute this extension of our econometric equation.

Since we have heteroskedasticity in our model, we decided to apply an econometric

is 14 percent.

Column (2) presents the full regression equation, given by Equation (51), estimated using OLS. The productivity and wage coe cients have positive signs and are signi cant

As discussed in Section 5.1 general equilibrium, Cournot, Hotelling and the Burdett and

have not corrected for this problem in our estimation.

5.7 Robustness checks

To further address the validity of the results we run two alternative regressions. First, we use a di erent type of productivity measure. Second, we use a di erent structural form on the estimated equation.

Variable	Productivity measure (1)	Structural form (2)
Productivity	0.0314* [0.0014]	0.0212* (0.0028)
Wage		0.0863* (0.0056)
Wage ²		0.0148* (0.0011)
Cons.	0.0134* [0.0007]	0.1023* 0.0050)
Adjusted R ²	0.0292	0.1938

Table 5: Robustness estimates

Dependent variable: Annual change in workers, in logs. *Signi cant at the 1 percent level. **Signi cant at the 5 percent level

Comparing column (1) in Table 5 and Table 4, we see that the labor productivity measure explains 14.01 percent while the TFP measure explains 2.9 percent of the variance in average annual worker growth. The signs of the coe cient in both table are the same and they are both signi cant at the 1 percent level. In conclusion, our productivity measure from Section 5.6 seems to be robust.

5.7.2 Structural form

We estimate an alternative model where the growth function F() is approximated by a second order logarithmic expansion. In order for our estimator (OLS) to be the best linear unbiased estimator an important factor is that the model is correctly specified, or in other words the model is the true functional form [33]. The hint for investigating this is our low $R^2 = 0.23$ as well as the conclusion from the Ramsey RESET¹⁸. We propose the following equation as an alternative to our main model

$$[\ln S_{it^{0}} \quad \ln S_{it}] = d = + {}_{1} \ln A_{it} + {}_{2} \ln w_{it} + {}_{3} \ln A_{it}^{2} + {}_{4} \ln w_{it}^{2} + {}_{5} \ln A_{it} \ln w_{it} + {}_{it}$$
(58)

The Ramsey RESET nds that in this equation there is no omitted variables. Since we found heteroskedasticity and multicollinearity in our main estimates we test for these

¹⁸Our model may have omitted variables or wrong structural form

problems. The conclusion from the BPG test is that there is homoskedasticity and the variance-infalting factor (VIF) test suggests there is multicollinearity. We remove the variables $\ln A_{it}$ $\ln w_{it}$ because of multicollinearity. In addition, we remove $\ln A_{it}^2$ since it is not signi cant. The resulting VIF-test ¹⁹ states that multicollinearity is no longer a problem and the Ramsey RESET still predict no missing variables.

The results are shown in column (2) in Table 5. We see that all variables are significant at the 1 percent level and the adjusted $R^2 = 0.1938$. These results are almost identical to the ones obtain with a rst-order logarithmic expansion. The adjusted R^2 is smaller than for our initial model. These results support our ndings.

6 Conclusion

The main objective of this paper is to investigate the relationship between productivity and worker ows. In addition, a brief investigation of selected stylized facts found in the literature is also conducted. We nd that the data shows the same stylized facts regarding growth, productivity, size and bankruptcy risk as found in the literature.

We nd that the size distribution of rms is highly skewed towards smaller rms. Productivity levels are quite dispersed, meaning that there is productivity di erences between rms. We nd suggestive evidence for a positive relationship between mean wage and mean productivity in our data. In addition, we nd a positive relationship between both the probability of survival (P < 0.05) and variability of growth (P < 0.01) with size and age as explanatory variables.

Our main nding is that there is a positive relationship between productivity and average annual worker growth (P < 0.01). This suggest that more productive rms hire workers faster. However, this does not necessarily suggest that the most productive rm is also the largest rm. But, given enough time this might be the case as well. We also nd that there is a positive relationship between wage (P < 0.01) and average annual worker growth. The wage-growth relationship is contrary to cost minimization, but in line with competitive search theory. Our results could therefore be seen as supportive evidence for the validity of competitive search models. Furthermore, we ind a negative relationship between growth and age (P < 0.01). We also ind a negative relationship between growth and size (P < 0.01), suggesting that Gibrat's Law fails.

¹⁹See Table 10 in Appendix C

The association we identify between productivity and average annual worker growth is robust. To deal with problems of measurement error we test our hypothesis using another measure for productivity, TFP. The use of TFP yields the same results, positive relationship between productivity and average annual worker growth (P < 0.01). In order to deal with problems of speci cation error we run a regression using a di erent structural form. The use of an alternative structural form yields the same results, positive relationship between productivity and average annual worker growth (P < 0.01).

In conclusion, we have found that there is a positive relationship between average annual worker growth and productivity in the Norwegian manufacturing industry suggesting that more productive rms attract workers faster than less productive rms. Given enough time, a fast growing small rm could eventually be larger than a slow growing large rm. Our ndings are in line with the microeconomic theories.

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A Mathematical Appendix

A.1 Counout Competition

The rms have two decisions. First the decision of how much output to produce, and then how to produced this output with the least amount of cost. We start with the output market decision rst. The rms compete in quantum in each period. The rms have the following pro t function [62]

$$i = q_i \quad (1 \quad q_i \quad q_j) \quad c_i \quad q \tag{59}$$

where $c_i q = C(w; r; q)$, just to simplify notation at the moment. The rst order condition for the optimization problem are:

$$\frac{@}{@q_i}(^{i}) \quad 2q_i + 1 \quad q_j \quad c_i = 0$$
(60)

$$\frac{@}{@q_i}(^{i}) \quad 2q_j + 1 \quad q_i \quad c_j = 0$$
(61)

By solving (60) and (61) for the respective quantities and substitution in to each other we obtain the following optimal quantities:

$$q_i = \frac{1 - 2c_i + c_j}{3}$$
(62)

$$q_j = \frac{1 - 2c_j + c_i}{3}$$
(63)

Equation (62) and (63) state that the optimal quantity is a function of the rms own production cost and the cost of the competition. The pro t of the rms are given by the following

$${}^{1} = \frac{(1 \quad 2C_{i} + C_{j})^{2}}{9}$$
(64)

The rms know what quantity to produce and need to decide how to produce this given amount of output for the least amount of resources. Suppose a long run cost minimization.

$$C(w; r; q) = \min_{K;L} r \quad K + w \quad L$$

such that $A_i K \quad L^1 = q$

Where $A_i > 0$ is a level speci c production technology. Solving the optimization we obtain the optimal demand for K and L

$$\mathcal{K} = q \quad \frac{1}{A_i} \quad \frac{W}{1 \quad r} \quad \frac{W}{r} \quad (65)$$

$$L = q \quad \frac{1}{A_i} \quad \frac{1}{W} \tag{66}$$

The cost function is de ned as:

$$C(W; r; q) = r \quad K \quad +W \quad L \tag{67}$$

combining the optimal demands with the cost function, we obtain

$$C(w;r;q) = \frac{1}{A_i} \prod_{i=1}^{n} \frac{1}{1} + \frac{1}{1} \prod_{i=1}^{n} \frac{r_i w^1}{q} \qquad (68)$$

$$C^{\emptyset}(W;r;q) = \frac{1}{A_{i}} \qquad \frac{1}{1} \qquad + \qquad \frac{1}{1} \qquad r \quad w^{1} \qquad (69)$$

By combining equation (62), (63) and the marginal cost, derived above, we get

$$q_{i} = \frac{1 + A_{j}^{1} 2A_{i}^{1}}{r w^{1}} + \frac{1}{1} + \frac{i}{1} r w^{1}}$$
(70)

$$q_{j} = \frac{1 + A_{j}^{1} 2A_{j}^{1} \frac{1}{1} + \frac{1}{1} + \frac{1}{1} r w^{1}}{3}$$
(71)

Which means that q_i is a function of A_i ; A_j ; :: r; w. We now see that the demand for input K and L is a function of $q; A_i$; :: r; w, such that we can write the demand for inputs as:

$$\mathcal{K} = F(q(A_i; A_j); A_i; ; r; w) \tag{72}$$

$$L = F(q(A_i; A_j); A_i; ; r; w)$$
(73)

C Tables Appendix

Variable	Obs.	Mean	Std. Dev.	Min	Max
Survival:					
Survival	8057	0.53	0.50	0	1
Log[Size]	8057	28.62	154.18	1	9094
Log[Age]	8057	13.62	13.94	1	158
Growth/var./reall.:					
Growth	2942	0.003	0.10	-0.52	0.83
Log[Std. of Gro.]	2942	0.21	1.25	-1.39	5.60
Log[Size]	2942	2.32	1.29	0	8.17
Log[Age]	2942	2.14	0	0	5.06
Log[Prod]	2942	-0.75	0.78	-6.45	4.98
Log[wage]	2942	-1.34	0.59	-7.7	4.47

Table 6: Summary Statistics of Logarithmic Variables

The table presents the logarithmic values used in the estimation

Year	PPI	Manufacturing
2000	100	100
2000	100	
2001	101.9	104.7
2002	97.8	110.6
2003	99.2	115.4
2004	102.3	120.2
2005	105.8	124.8
2006	109.0	130.0
2007	113.8	138.2
2008	122.7	145.9

The table show the de ation indexes collected from the OECD. PPI is producer price index.

Year	3 year gov. bond	-COe .	Risk prem.	CAPM	Depr.	User price
2000	6.61	1	6	12.61	14.9	27.51
2001	6.44	1	6	12.44	14.9	27.34
2002	6.39	1	6	12.39	14.9	27.29
2003	4.24	1	6	10.24	14.9	25.14
2004	2.95	1	6	8.95	14.9	23.85
2005	2.90	1	6	8.9	14.9	23.8
2006	3.74	1	6	9.74	14.9	24.64
2007	4.79	1	6	10.79	14.9	25.69
2008	4.53	1	6	10.53	14.9	25.43

Table 8: Calculation of user price of capital (capital cost)

Table 9: Variance-in ating factor (VIF) test from the initial estimation

Variable	VIF	1/VIF
In_size In_prod In_wage In_age	1.10 2.32 2.30 1.07	0.9110 0.4305 0.4351 0.9381
Mean VIF	1.70	

The table show the VIF values of our independent variables.

Variable	VIF	1/VIF
In_wage In_wage2 In_prod	5.46 3.63 2.22	0.1833 0.2758 0.4511
Mean VIF	3.77	

 Table 10: Variance-in ating factor (VIF) test for alternative structural form

The table show the VIF values of our independent variables.

.,		Robust		
Variable	Coe cients	Std. Error	t-statistic	Prob.
	0.0000	0.0017	14.04	0.000
In_prod	0.0238	0.0017	14.21	0.000
In_wage	0.0367	0.0030	12.28	0.000
In_age	-0.0111	0.0006	-17.10	0.000
In_size	-0.0158	0.0005	-32.51	0.000
D2	-0.0130	0.0044	-2.95	0.003
D3	-0.0274	0.0073	-3.74	0.000
D4	0.0112	0.0029	3.84	0.000
D5	-0.0492	0.0057	-8.61	0.000
D6	-0.0274	0.0030	-9.21	0.000
D7	0.0183	0.0031	5.99	0.000
D8	-0.0206	0.0054	-3.85	0.000
D9	-0.0029	0.0032	-0.89	0.375
D10	0.0018	0.0033	0.55	0.582
D11	0.0198	0.0058	3.39	0.001
D12	-0.0074	0.0029	-2.53	0.011
D13	-0.0130	0.0030	-4.27	0.000
D14	-0.0285	0.0154	-1.85	0.065
D15	0.0015	0.0035	0.44	0.658
D16	0.0320	0.0078	4.09	0.000
D17	-0.0042	0.0038	-1.12	0.265
D18	-0.0138	0.0052	-2.67	0.008
D19	0.0009	0.0036	0.25	0.802
D20	-0.0144	0.0031	-4.61	0.000
D21	0.0056	0.0066	0.85	0.395
Constant	0.1379	0.0046	29.67	0.000

 Table 11: Regression results from the xed e ects model

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Dependent variable: Annual change in workers, in logs. D1 is the reference industry, NACE 17.

D Data Appendix

D.1 Source of government bond

Norges Bank:

Statsobligasjoner. Annual average of daily quotations. The basis is the 3 year. http://www.norges-bank.no/templates/article____55495.aspx